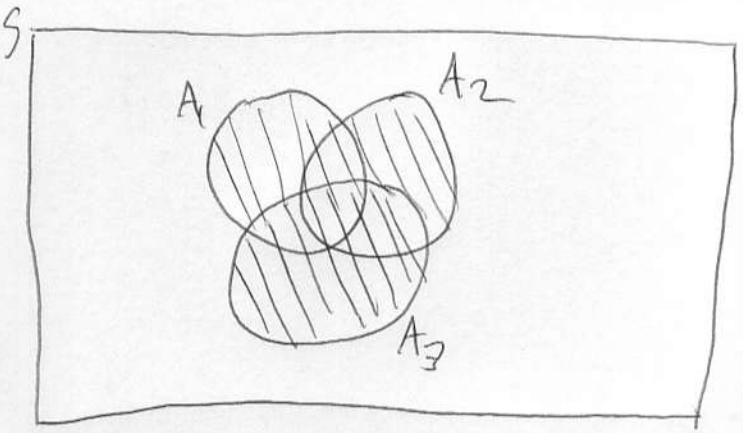
HW #2

CH2: 8, 12, 19, 31, 40, 45, 63, 82, 106

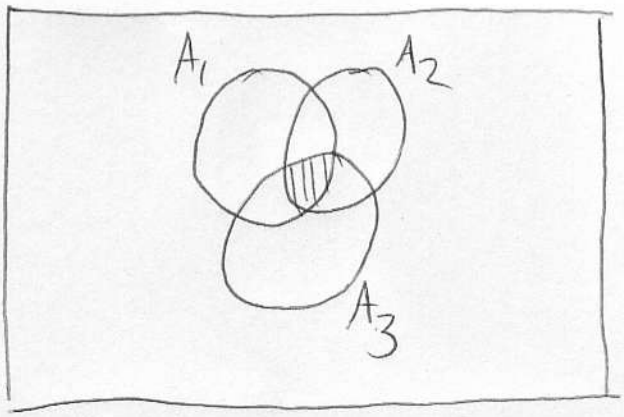
Problem 8

A_i = "Plant i completed by the contract date"

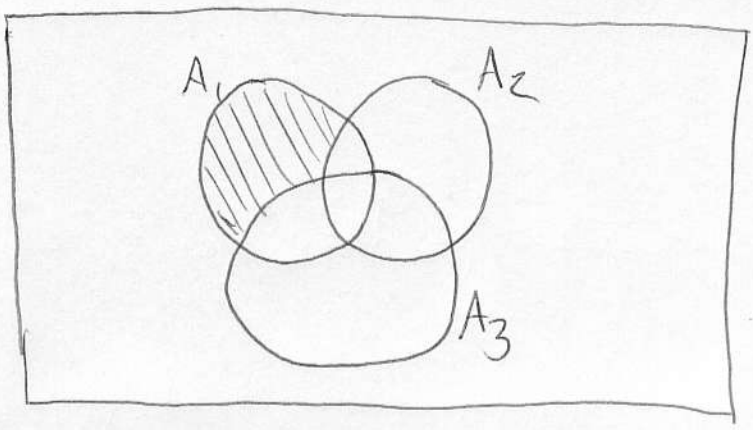
a) $A_1 \cup A_2 \cup A_3$



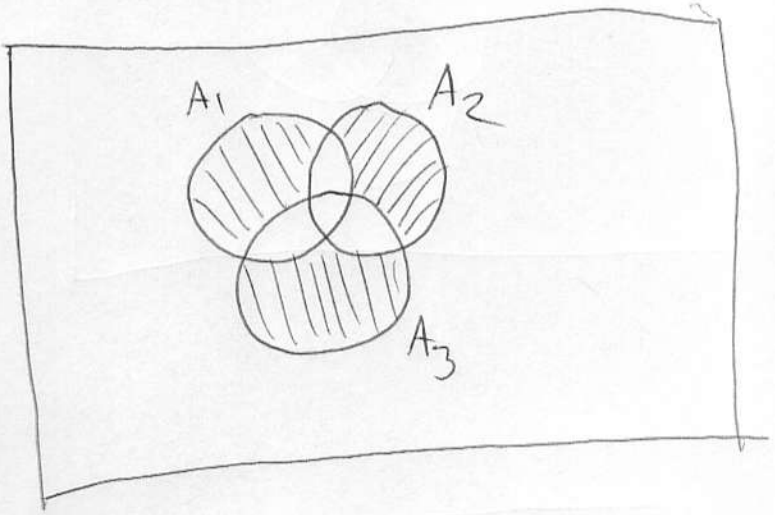
b) $A_1 \cap A_2 \cap A_3$



c) $A_1 \cap (A_2 \cup A_3)^c$

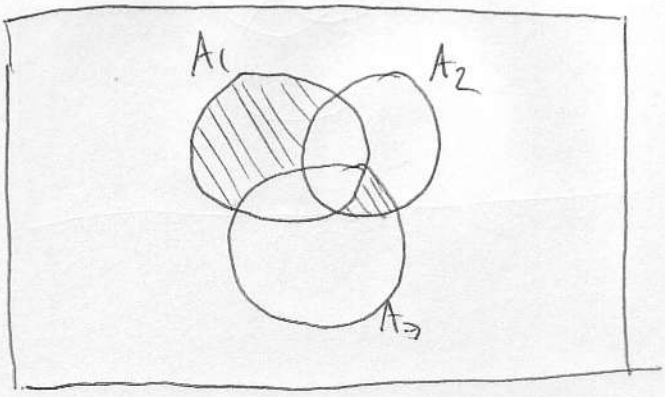


d) $(A_1 \cap (A_2 \cup A_3)^c) \cup (A_2 \cap (A_1 \cup A_3)^c) \cup (A_3 \cap (A_1 \cup A_2)^c)$



$(A_1 \cap (A_2 \cup A_3)^c) \cup ((A_2 \cap A_3) \cap A_1^c)$

e



Problem 12

A = "has Visa credit card"

B = "has MasterCard credit card"

$P(A) = .5 \quad P(B) = .4 \quad P(A \cap B) = .25$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .5 + .4 - .25$

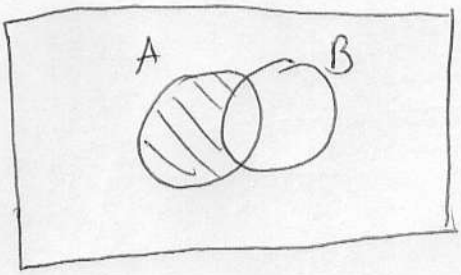
$= .65$

(b) $P(\text{neither card}) = P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - .65$

$= .35$

(c)



$A \cap B^c \qquad P(A \cap B^c) = ?$

$(A \cap B^c) \cup (A \cap B) = A$

$\Rightarrow P(A \cap B^c) + P(A \cap B) = P(A)$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$= .5 - .25 = .25$

Problem 19

$$P(A) = \frac{724}{10000} = .0724 \qquad P(B) = \frac{751}{10000} = .0751$$

$$P(A \cup B) = \frac{1159}{10000} = 0.1159 \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= .0724 + .0751 - .1159$$
$$= .0316$$

a) Want $P((A \cup B)^c) = 1 - P(A \cup B)$
$$= 1 - 0.1159$$
$$= .8841$$

b) Want $P(B \cap A^c) = P(B) - P(A \cap B)$
$$= .0751 - .0316$$
$$= .0435$$

Problem 31

a) $9 \times 27 = 243$

(9 ↑ Beethoven, 27 ↑ Mozart)

b) $9 \times 27 \times 15 = 3645$ programs

(9 ↑ Beethoven, 27 ↑ Mozart, 15 ↑ Schubert)

Assume 365 days a year
Thus 9 years 360 days
Is approx a decade (10 years)

Problem 40

a) If all molecules distinguishible then there is $12! = {}^{12}P_{12}$ ways to order them.

Because there are ${}^{3}P_3 = 3!$ ways to order each of the 3 A's, 3 B's, 3 C's and 3 D's there is $12!/(3!3!3!3!) = 369600$

(b) treat the (groups of) 3 A's, 3 B's, 3 C's and 3 D's as individual blocks so there is

4 ways to choose first block of three
3 ways to choose second block of three
2 " " " third " " "
1 ways to choose fourth block of three

12

$\#\text{molecules} = 4*3*2*1 = 4!$

So probability $= \dfrac{4!}{(12!/(3!3!3!3!))} = \dfrac{24}{369600}$

Problem 45

A = "type A selected" C = "ethnic group 3 selected"
B = "type B selected"

a) $P(A) = .106 + .141 + .200$
$= .447$

$P(C) = .215 + .200 + .065 + .020$
$= .5$

$A \cap C$ = "type A and ethnic group 3"

$P(A \cap C) = .200$

b) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.2}{.5} = .4$

↑ probability of having type A given that we know individual is ethnic group 3

$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{.2}{.447} = .447$

↑ Probability of being from ethnic group 3 given that we know individual has type A

c) $P(\text{ethnic group 1} \mid \text{not type B})$

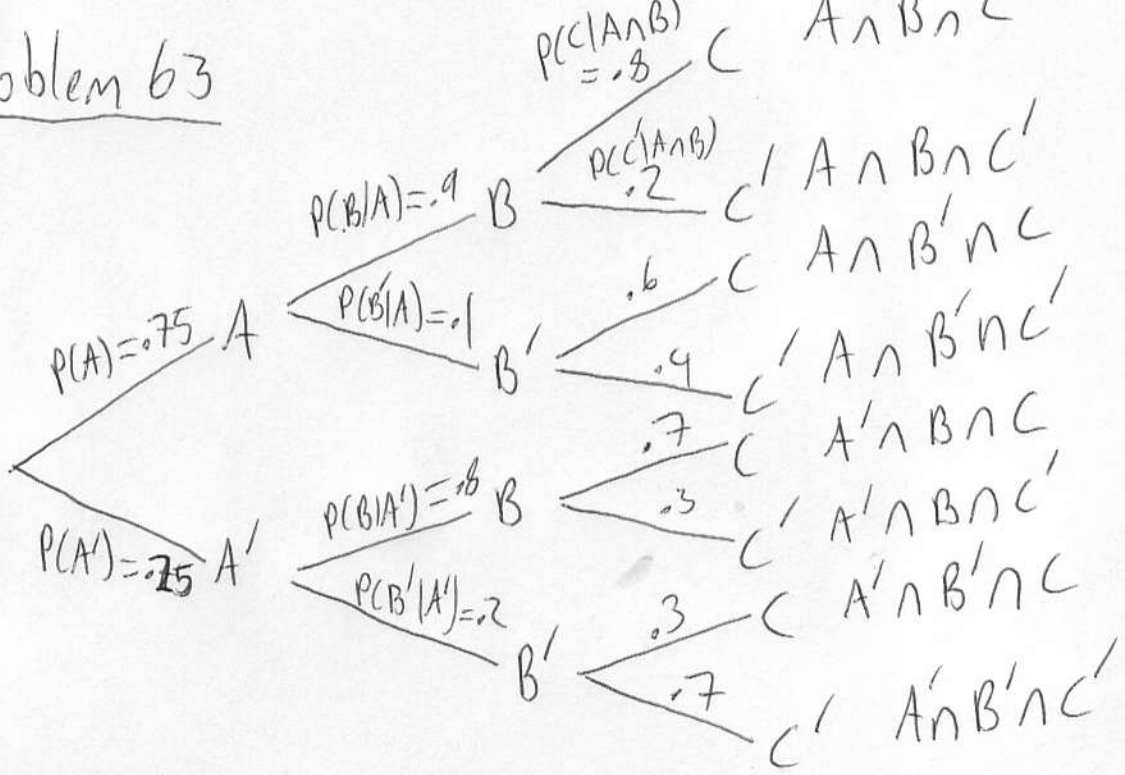
$$= \frac{P(\text{ethnic group 1 and not type B})}{P(\text{not type B})}$$

$$= \frac{.082 + .106 + .004}{1 - (.008 + .018 + .065)}$$

$$= .211$$

Problem 63

a)



b) $P(A \cap B \cap C) = P(C \mid A \cap B) * P(B \mid A) P(A)$

$$= .8 * .9 * .75 = .54$$

c) $P(B \cap C) = P(B \cap C \mid A) P(A) + P(B \cap C \mid A') P(A')$

$$= .8 * .9 * .75 + .8 * .7 * .25 = .68$$

d) $P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B')P(A\cap B')$
$+ P(C|A'\cap B)P(A'\cap B) + P(C|A'\cap B')P(A'\cap B')$

$= .75*.9*.8 + .75*.1*.6$
$+ .25*.8*.7 + .25*.2*.3$

$= .74$

e) $P(A|B\cap C) = \frac{P(A\cap B\cap C)}{P(B\cap C)} = \frac{.54}{.68} = .794$

Problem 82

P(individual vehicle passes) = 0.7

a) P(all three pass) = $(.7)(.7)(.7) = .343$

b) P(at least one fails) = 1 − P(none fail)
$= 1 - (.7)(.7)(.7) = .657$

c) P(exactly one passes) = $_3C_1 (.7)(.3)^2 = .189$

↑ #ways to choose car that passes

d) P(at most one passes)

$= P(\text{None Pass}) + P(\text{One Passes})$

$= (.3)(.3)(.3) + {}_3C_1(.7)(.3)^2 = .216$

e) $P(\text{all three pass} \mid \text{at least one passed}) = \dfrac{P(\text{all three passed and at least one passed})}{P(\text{at least one passed})}$

$= \dfrac{.343}{1-.3^3} = .35$

Problem 106

End when have reached either 3 strikes or 4 balls

3 strikes = strikeout
4 balls = walk

B - Ball
S - strike

a) $P(\text{walk on 4th pitch}) = P(BBBB) = (.5)^4 = .0625$

b) $P(\text{walk on 6th pitch}) = {}_5C_2(.5)^6 = 10(.5)^6 = .15625$

↑ # ways to choose the two strikes

c) $P(\text{walks}) = P(\text{walk on 4th pitch}) + P(\text{walk on 5th pitch}) + P(\text{walk on 6th pitch})$

P(walk on 4th pitch) $= (.5)^4 = .0625$

P(walk on 5th pitch) $= {}_4C_1 (.5)^5 = 4(.5)^5 = .125$

P(walk on 6th pitch) $= .15625$

P(walk) $= .0625 + .125 + .15625 = .34375$

d) For first batter up to score with noone else out need 4 batters in a row to walk

$(.34375)^4 = .014$